Dynamic Vibration Analysis for Non-linear Partial Differential Equation of the Beam - columns with Shear Deformation and Rotary Inertia by AGM

M.R. Akbari[1], D.D. Ganji[2*], A.R. Goltabar[3]

[1] Department of civil engineering and chemical engineering University of Tehran, Iran
[2*] Department of Mechanical Engineering, Babol University of Technology, P.O. Box 484, Babol, Iran
[3] Department of Civil Engineering Bbol University of Technology, Babol, Iran

ddg_davood@yahoo.com

Abstract

In this study, nonlinear partial differential equations governing vibrating beam- columns with variable cross-section of the new method is applied to solve. Our purpose is to enhance the ability of solving the mentioned nonlinear differential equation with a simple and innovative approach which was named Akbari-Ganji's Method or AGM. Comparisons are made between AGM and Numerical Method (Runge-Kutte 4th). The results reveal that this method is very effective and simple, and can be applied for other nonlinear problems. It is necessary to mention that there are some valuable advantages in this way of solving nonlinear differential equations and also most of the sets of differential equations can be answered in this manner which in the other methods they have not had acceptable solutions up to now. We can solve equation(s) and consequently there is no need to utilize similarity solutions which makes the solution procedure a time-consuming task. According to the afore-mentioned assertions which will be proved in this case study, the process of solving nonlinear equation(s) will be very easy and convenient in comparison with the other methods. According to the explanations given about the capabilities of this method (AGM), non-linear partial differential equations governing the vibration of the beam-column are investigated for dynamic systems.

Keywords

New Method; Akbari-Ganji's Method (AGM); Oscillating Systems; Beam–columns; Angular Frequency

Introduction

Many physical phenomena are modeled by nonlinear differential equations in order to have more opportunities to control the real objects in the universe, for example, vibration of structural systems associated with nonlinear properties in civil engineering is considered to be in this category. To better understand, it is better to say that nonlinear oscillator models have widely been used in many areas of physics and engineering and play an important role in mechanical and structural dynamics (eg, earthquakes) for the comprehensive understanding and accurate prediction of motion. In accordance with the afore-mentioned explanations like in this case study, attempts have been made in order to solve this kind of problems which have attracted much attention in the process of weaving and also to find their suitable solutions. It is noteworthy that many of the classic analytical methods such as traditional perturbation techniques which were presented for solving nonlinear equations not only have many shortcomings, but also not valid for strongly nonlinear differential equations. Since, a minority of scientific problems have precise analytical solutions and most of the afore-mentioned methods do not have the capability to solve nonlinear differential equations easily and accurately so in this case study in order to finish this difficulty, a new approach (AGM) for solving complicated nonlinear differential equations in various fields of study especially in mechanical and civil engineering has been presented.

In this paper, the solution of a nonlinear partial differential equation of a beam-column which deforms due to a shear force and a rotational inertia is investigated. In this analysis, the effects of small ratio of length to depth of the beam cause a considerable changing on the dynamical and vibrational response of the beam. Figure 2 obviously depicts the effects of shear force and inertial resistance on rotational acceleration of the beam cross section. As a result, in condition that the ratio of depth to span is smaller, two factors play an important role in dynamic response of beam-column. The first factor is deformation which

due to shear force and the second factor is inertia resistance against the rotational acceleration of beam cross section. According to Fig. 2. two components of displacement of beam cross section are as follows:

-Rotational inertia (m_I) will be caused by rotational section of beam (angle α) from it's main condition the perpendicular condition.

-Deformation that due to the shear stress (by assumption that beam cross section also remain as a plate) will make angle β.

As it has been stated that this theory has been utilized in fields of mechanical and civil engineering specially in earthquakes analysis. Also, such beams (ie beams connected) can be used in structural steel and concrete.

Nomenclature

L	Length of the beam – column
E	Young's modulus
N	Axial force
m	Mass per unit length of beam – column
k	Shear modulus cross beam – column (For rectangular k = 5/6)
A	Cross - section beam – column
G	Young's modulus, shear
q(x)	Wide load on the beam – column
c	Damper coefficient
r	Radius of gyration of the cross – section($\sqrt{\dfrac{I}{A}}$)
I	Moment of inertia
U=U(x,t)	Vibrating beam displacement
ϕ	Shape modal
z(t)	Displacement function of time
β	shear deformation angle
α	RotationInertia angle
m_I	Rotational inertia per unit length of the beam – column

The analytical Method

In general, vibrational equations and their initial conditions are defined for different systems as follows:

$$f(\ddot{u}, \dot{u}, u, F_0 \sin(\omega_0 t)) = 0 \qquad (1)$$

$$\{u(0) = A, \dot{u}(0) = 0\} \qquad (2)$$

Choosing the Answer of the Governing Equation for Solving Differential Equations by AGM

In AGM, a total answer with constant coefficients is required in order to solve differential equations in various fields of study such as vibrations, structures, fluids and heat transfer. In vibrational systems with respect to the kind of vibration, it is necessary to choose the mentioned answer in AGM. To clarify here, we divide vibrational systems into two general forms:

1) Vibrational Systems without any External Force

Differential equations governing on this kind of vibrational systems are introduced in the following form:

$$f(\ddot{u}, \dot{u}, u) = 0 \qquad (3)$$

Now, the answer of this kind of vibrational system is chosen as:

$$u(t) = e^{-bt}\{A\cos(\omega t) + B\sin(\omega t)\} \qquad (4)$$

According to trigonometric relationships, Eq.(4) is rewritten as follows:

$$u(t) = e^{-bt}(a\cos(\omega t + \varphi)) \qquad (5)$$

It is notable that in the above equation $a = \sqrt{A^2 + B^2}$ and $\varphi = \arctan(\dfrac{B}{A})$.

Sometimes for increasing the precision of the considered answer of Eq.(3), we are able to add another term in the form of cosine by inspiration of Fourier cosine series expansion as follows:

$$u(t) = e^{-bt}\{a\cos(\omega t + \varphi) + d\cos(2\omega t + \varphi)\} \qquad (6)$$

In the above equation, we are able to omit the term (e^{-bt}) to facilitate the computational operations in AGM if the system is considered without any damping components.

Generally speaking in AGM, Eq.(5) or Eq.(6) is assumed as the answer of the vibrational differential equation (3) that its constant coefficients which are a, b, d, ω (angular frequency) and φ (initial vibrational phase) can easily be obtained by applying the given initial conditions in Eq.(2). And also the above procedure will completely be explained through the presented example in the foregoing part of the paper.

It is noteworthy that if there is no damping component in the vibrational system, the constant coefficient of b in Eq.(5) and Eq.(6) will automatically be computed zero in AGM solution procedure.

On the contrary, the parameter b in Eq.(5) and Eq.(6) for the other kind of vibrational system with damping component is obtained as a nonzero parameter in AGM.

2) Vibrational Systems with External Force

In this step, it is assumed that the external forces exerted on the vibrational systems are defined as:

$$F(t) = F_0 \sin(\omega_0 t) \qquad (7)$$

As a result, the differential equation governing on the vibrational system is expressed like Eq.(1) as

follows:

$$f(\ddot{u}, \dot{u}, u, F_0 \sin(\omega_0 t)) = 0 \qquad (8)$$

The answer of the above equation is introduced as the sum of the particular solution (u_p) and the harmonic solution (u_h) as follows:

$$u_h(t) = e^{-bt}\{A\cos(\omega t) + B\sin(\omega t)\};$$
$$u_p(t) = M\cos(\omega_0 t) + N\sin(\omega_0 t) \qquad (9)$$

Then

$$u(t) = u_p + u_h \qquad (10)$$

By utilizing trigonometric relationships and substituting the yielded equations into Eq.(10), the desired answer will be obtained in the form of:

$$u(t) = e^{-bt}\{a\cos(\omega t + \varphi)) + d\cos(\omega_0 t + \phi)\} \qquad (11)$$

In order to increase the precision of the achieved equation, we are able to add another term in the form of cosine by inspiration of Fourier cosine series expansion as follows:

$$u(t) = e^{-bt}\{a.\cos(\omega t + \varphi) + c.\cos(2\omega t + \varphi)\} + d.\cos(\omega_0 t + \phi) \qquad (12)$$

Finally, the exact solution of the all vibrational differential equations can be obtained in accordance with the following equation:

$$u(t) = e^{-at}\{\sum_{k=1}^{\infty} b_k \cos(k\omega t + \varphi_k)\} + d\cos(\omega_0 t + \phi) \qquad (13)$$

To deeply understand the above procedure, reading the following lines is recommended.

Since the constant coefficient (b) in vibrational systems without damping components is always obtained zero, we can add the term $(b.t)$ instead of (e^{-bt}) in Eq.(12) to decrease computational operations in the following form:

$$u(t) = (b.t) + a.\cos(\omega t + \varphi) + d.\cos(2\omega t + \varphi) \qquad (14)$$

Based on the above explanations, by applying initial conditions on a system without damping component, the value of parameter (b) is always zero for Eq.(12) and Eq.(13). Therefore the role of parameter (b) in the both of Eq.(12) and Eq.(13) each of which can be considered as the answer of the vibrational problems is individually considered as a catalyst for increasing the precision of the assumed answer. However according to Eq.(16) and Eq.(20) after applying initial conditions on the vibrational system in both states (with external force and without external force) by AGM, the value of parameter (b) is computed zero because the mentioned system has a free vibration without any damping component.

Again, it is mentioned that in order to decrease computational operations for systems without damping component and since we know that (b) in the term (e^{-bt}) is zero so (e^{-bt}) can be omitted from Eq.(11). Consequently, Eq.(11) which has been considered as the answer of the systems without any damping component can be rewritten as follows:

$$u(t) = a.\cos(\omega t + \varphi) + d.\cos(\omega_0 t + \phi) \qquad (15)$$

The constant coefficients of Eq.(11) or Eq.(12) which are $\{a, b, \omega, \varphi, c, d, \phi\}$ will easily be computed in AGM by applying the initial conditions of Eq.(2).

Application of Initial Conditions to Compute Constant Coefficients and Angular Frequency by AGM

In AGM, the application of initial conditions of Eq.(2) is done in the two following forms:

1) Applying the Initial Conditions on the Answer of Differential Equation

In regard to the kind of vibrational system (with external force and without external force) which was completely discussed in the previous part of this case study, a function is chosen as the answer of the differential equation from Eq.(5) or Eq.(6) for the systems without external forces and from Eq.(11) or Eq.(12) for the defined systems with external forces and then the initial conditions are applied on the selected function as follows:

$$u(t) = u(IC) \qquad (16)$$

It is notable that IC is the abbreviation of introduced initial conditions of Eq.(2).

2) Appling the Initial Conditions on the Main Differential Equation and Its Derivatives

After choosing a function as the answer of differential equation according to the kind of vibrational system, this is the best time to substitute the mentioned answer into the main differential equation instead of its dependent variable (u).

Assume the general equation of the vibration such as Eq.(1) with time-independent parameter (t) and dependent function (u) as:

$$f(\ddot{u}, \dot{u}, u, F_0 \sin(\omega_0 t)) = 0 \qquad (17)$$

Therefore, on the basis of the kind of vibrational system, a function as the answer of the differential equation such as Eq.(5) or Eq.(6) and Eq.(11) or Eq.(12) are considered as follows:

$$u = g(t) \qquad (18)$$

In this step, the afore-mentioned equation is substituted into Eq.(17) instead of (u) in the following form:

$$f(t) = f(g''(t), g'(t), g(t), F_0 \sin(\omega_0 t)) \qquad (19)$$

Eventually, the application of initial conditions on Eq.(19) and its derivatives is expressed as:

$$
\begin{aligned}
&f(IC) = f(g''(IC), g'(IC), g(IC), ...), \\
&f'(IC) = f'(g''(IC), g'(IC), g(IC), ...), \\
&f''(IC) = f''(g''(IC), g'(IC), g(IC), ...), \\
&.....
\end{aligned} \qquad (20)
$$

To end up, it is better to say that in AGM after applying the initial conditions on Eq.(18), Eq.(19) and also on its derivatives from Eq.(20) according to the order of differential equation and utilizing the own two given initial conditions of Eq.(2), a set of algebraic equations which consists of n equations with n unknowns is created. Therefore, the constant coefficients (a, b, c, d, angular frequency "ω" and initial phase) are easily achieved which this procedure will thoroughly be explained in the form of an example in the foregoing part of this paper.

It is noteworthy that in Eq.(20), we are able to use the derivatives of $f(t)$ with higher orders until the number of yielded equations is equal to that of the mentioned constant coefficients of the assumed answer.

Theorical Formulation

Consider a beam-columns with non-uniform cross-section according to the following figure:
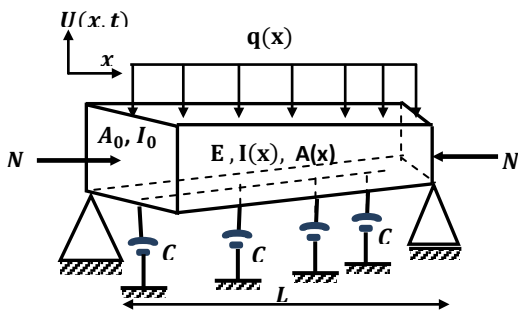
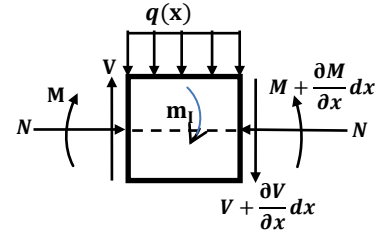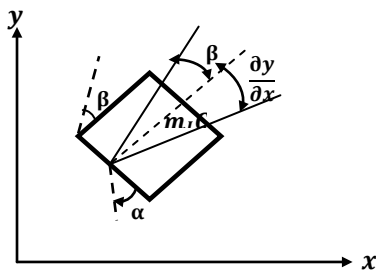FIG. 1 THE SCHEMATIC DIAGRAM OF THE PHYSICAL MODEL.

FIG. 2 BEAM - COLUMN WITH THE EFFECTS OF SHEAR DEFORMATION AND ROTARY INERTIA AND EQUALFORCES

According to the Fig. 2, the balance of forces on the element (beam-column) the vibration differential equation can be obtained as follows [5,8,9]:

$$
\begin{aligned}
&\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 U}{\partial x^2}\right) + c\frac{\partial U}{\partial t} + N\frac{\partial^2 U}{\partial x^2} - \left[q(x) - m\frac{\partial^2 U}{\partial t^2}\right] \\
&-mr^2\frac{\partial^4 U}{\partial x^2 \partial t^2} + \frac{EI}{kAG}\frac{\partial^2}{\partial x^2}\left[q(x) - m\frac{\partial^2 U}{\partial t^2}\right] \\
&-\frac{EI}{2L}\left(\frac{\partial^2 U}{\partial x^2}\right)\int_0^L \left(\frac{\partial U}{\partial t}\right)^2 dx = 0
\end{aligned} \qquad (21)
$$

The beam cross-section moment of inertia (I) and cross-section (A) at along the beam length, we consider Function as the following:

$$A = A_0(1 - \varepsilon x)^2, I = I_0(1 - \varepsilon x)^4 \qquad (22)$$

A_0 and I_0 is the basic beam-colum characteristics and ε is Constant.

Using separate equations $U = \phi(x).z(t)$ and put in a non-linear differential Eq. 21, proceeds obtained as follows[10,11]:

$$
\begin{aligned}
&\frac{1}{kGA_0\phi}\{(2E\varepsilon I_0 x + kGA_0 - r^2 kGA_0 - EI_0 - E\varepsilon^2 I_0 x^2)\phi''\}\ddot{z} \\
&+\frac{c}{m}\dot{z} + \frac{1}{m\phi}\{EI_0(1-\varepsilon x)^4\phi^{(IV)} + 8EI_0\varepsilon(\varepsilon^3 x^3 - 3\varepsilon^3 x \\
&-3\varepsilon^2 x^2 + 3\varepsilon x - 1)\phi''' + (N + 12E\varepsilon^2 I_0 + 12EI_0\varepsilon^4 x^2)\phi''\}z \\
&-\{\frac{EA_0\phi''\int_0^L(\phi')^2 dx}{2mL\phi}\}z^3 + \frac{EI_0\varepsilon^2 x^2\phi'' - kA_0 G q(x)}{mkGA_0\phi} = 0
\end{aligned} \qquad (23)
$$

And applying the Galerkinmethod[12,13], the Eq. 23 is obtained as follows :

$$f(t) = \alpha \ddot{Z} + \beta \dot{Z} + \gamma Z - \lambda Z^3 + \eta = 0 \qquad (24)$$

Where dot denotes differentiation with respect to time and $\alpha, \beta, \gamma, \lambda, \eta$ are as follows:

$$
\begin{aligned}
\alpha = &\frac{1}{kGA_0\int_0^L \phi^2 dx}\Big[\int_0^L (2EI_0\varepsilon x + kGA_0 - r^2 kGA_0 \\
&- EI_0 - E\varepsilon^2 I_0 x^2)\phi''\phi dx\Big]
\end{aligned} \qquad (25)
$$

$$\beta = \frac{c}{m} \qquad (26)$$

$$\lambda = \frac{EI_0}{2mL\int_0^L \phi^2 dx}\{\phi''\int_0^L(\phi')^2 dx\}dx \qquad (28)$$

$$\eta = \frac{1}{m\,k\,GA_0 \int_0^L \phi^2 dx} \int_0^L \{ EI_0\,\varepsilon^2\,x^2\,\phi^{''} - kA_0 G\,q(x)\}\phi\,dx \quad (29)$$

In the above equations, ϕ is the modal shape function of the beam-column [14,15] and the center of the beam-column subjected to the following initial conditions at differential equation (24) are expressed in the forms of:

$$\zeta(0) = A, \dot{\zeta}(0) = 0 \quad (30)$$

Solving the Differential Equation with AGM

On the basis of the given explanations in the previous section, the answer of Eq.(24) in AGM is considered as follows:

$$z(t) = e^{-at}\{ b\cos(\omega.t + \varphi)\} \quad (31)$$

In AGM, the constant coefficients of Eq.(31) which are ω (angular frequency), a, b and φ (initial vibrational phase) can easily be computed by applying initial conditions according to the physical aspects of the problem.

Applying Initial or Boundary Conditions in AGM

The constant coefficients of Eq. (31) can be gained by applying the initial or boundary conditions in this new approach.

Due to the proposed physical model, there are no boundary conditions so the constant coefficients of Eq. (31) are obtained just with respect to the given initial conditions which have been presented in Eq. (24).

It is noteworthy that in the proposed method, initial or boundary conditions are applied in 2 ways as follows:

1) In general, the initial conditions are applied on Eq. (31) in the form of:

$$\theta = \theta(IC) \quad (32)$$

For simplicity, IC is considered as the abbreviation of the initial conditions.

As a result, applying the initial conditions on Eq.(31) is done as:

$$z(0) = A \text{ so } b\cos\varphi = A \quad (33)$$
$$z'(0) = 0 \text{ therefore } b(a\cos\varphi + \omega\sin\varphi) = 0 \quad (34)$$

2) The application of initial or boundary conditions on the main differential equation which in this case is Eq.(24) and also on its derivatives is done in the following general form:

$$f(z(t)) \rightarrow f(z(IC)) = 0, f'(z(IC)) = 0 ,\dots.. \quad (35)$$

Therefore, after substituting Eq.(31) which has been considered as the answer of the main differential equation into Eq.(24), the initial conditions are applied

on the obtained equation and also on its derivatives on the basis of Eq.(31) as follows:

$$f(z(0)): b(\alpha\,a^2 - \alpha\,\omega^2 - \beta a + \gamma\,\beta)\cos(\varphi)$$
$$+ b\,\omega(2\alpha a - \beta)\sin(\varphi) - \lambda\,b^3\cos^3(\varphi) + \eta = 0 \quad (36)$$

Then for the first derivative of the obtained equation, we have:

$$f'(z(0)) = 0$$
$$b(3\alpha a\omega^2 - \alpha a^3 + \beta a^2 - \beta\omega^2 + \gamma\,a)\cos(\varphi)$$
$$+ b\,\omega(\alpha\,\omega^2 - 3\alpha a^2 + 2a\beta - \gamma)\sin(\varphi)$$
$$+ 3\lambda b^3(a\cos\varphi + \omega\sin\varphi)\cos^2\varphi = 0 \quad (37)$$

By solving a set of algebraic equations which is consisted of four equations with four unknowns from Eqs.(33, 34) and Eqs. (36, 37), the constant coefficients a, b, φ and ω from Eq.(31) can easily be yielded as follows:

To simplify, the following new variables are introduced as:

$$\psi = 4\gamma A\alpha + 4\eta\alpha - 4\lambda A^3\alpha \quad (38)$$

Therefore, the constant coefficients of Eq.(27) are achieved as follows:

$$a = \frac{1}{2}\frac{\beta}{\alpha}, b = A\sqrt{\frac{\psi}{\psi - \beta^2 A}} \quad (39)$$

And with respect to the above procedure, ω (angular frequency), φ (initial vibrational phase) can be computed in the forms of:

$$\omega = \frac{1}{2\alpha}\sqrt{\frac{\psi - \beta^2 A}{A}}, \varphi = tg^{-1}\{\sqrt{\frac{\psi - \beta^2 A}{\psi}}\} \quad (40)$$

After substituting the obtained values from Eq.(39) and Eq.(40) into Eq.(31), the solution of the mentioned problem will be obtained as follows:

$$z(t) = A\sqrt{\frac{\psi}{\psi - \beta^2 A}}\, e^{-\frac{1}{2}\frac{\beta}{\alpha}t}\cos\{\frac{1}{2\alpha}\sqrt{\frac{\psi - \beta^2 A}{\psi}}t$$
$$+ tg^{-1}(\sqrt{\frac{\psi - \beta^2 A}{\psi}})\} \quad (41)$$

By selecting the physical values below:

$$A = 0.1, \alpha = 1.8, \beta = 0.1, \gamma = 0.1,$$
$$\eta = 0.00002, \lambda = 0.3 \quad (42)$$

the solution of Eq.(31) and the angular frequency which in this case study have been obtained by Eq.(41) and Eq.(42) are rewritten, respectively as follows:

$$\omega = 0.2307\,(\frac{Rad}{\sec}) \quad (43)$$
$$z(t) = 0.100722\,e^{-0.027778t}\cos(0.2307t - 0.11982)$$

Then, the charts of the obtained solution and its phase plane are drawn in the following:
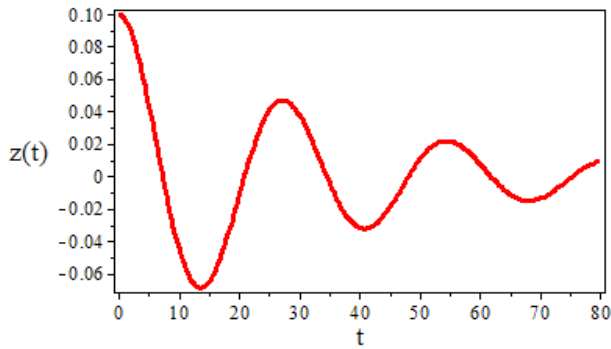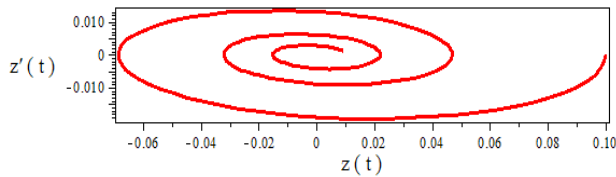
FIG. 3 THE OBTAINED SOLUTION BY AGM
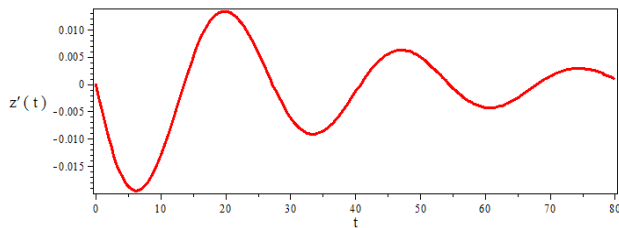

FIG. 4 THE RESULTED PHASE PLANE BY AGM

FIG. 5 THE DERIVATIVE OF THE OBTAINED
SOLUTION BY AGM

Numerical Solution

With regard to the given physical values from Eq. (40) and the determined domain $t \in \{0, 80\}$ which is defined in terms of second $(\sec)$, the solution of the differential equation is expressed numerically in the following table:

TABLE 1 THE OBTAINED RESULTS FOR $z(t)$ AND ITS DERIVATIVE
ACCORDING TO THE GIVEN PHYSICAL VALUES.

t	0.0	16	32	48	64	80
Z(t) Num	0.1000	-0.0582502	0.0205697	-0.0017143	-0.0107928	0.01021875
Z(t) AGM	0.1000	-0.058702	0.0230721	-0.010931	-0.008300	0.0096100
Percent error Num, AGM	0.0000	0.00776	0.1216	1.637	0.232	0.069
$\frac{dz(t)}{dt}$, Num	0.0000	0.0081833	-0.0089258	0.0061275	-0.0028106	0.0004500
$\frac{dz(t)}{dt}$, AGM	0.0000	0.078417	-0.0085774	0.0061506	-0.0032000	0.0005010
Percent error Num, AGM	0.0000	0.0417	0.0397	0.00337	0.138	1.156

Comparing the Achieved Solutions by Numerical Method and AGM

Due to the obtained solution from Eq. (41) by AGM, which is Eq.(43), and the obtained Numerical Solution whose results have been presented in table 1, we have the following comparisons:
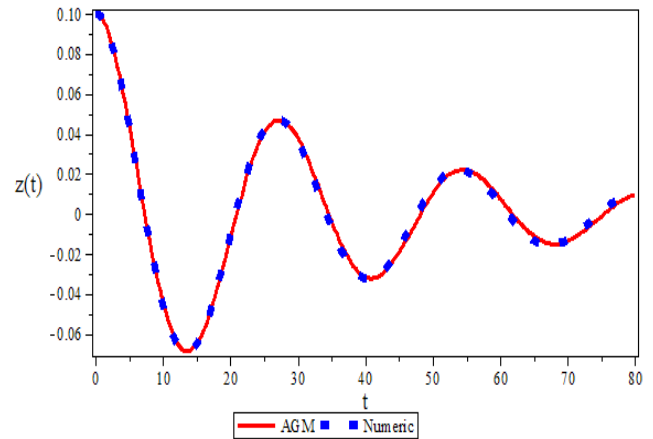
FIG. 6 A COMPARISON BETWEEN THE ACHIEVED SOLUTIONS
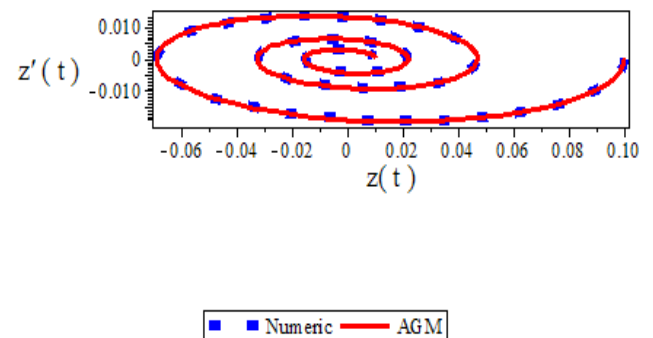BY AGM AND NUMERICAL METHOD.

FIG. 7 COMPARING THE RESULTED PHASE PLANES BY AGM
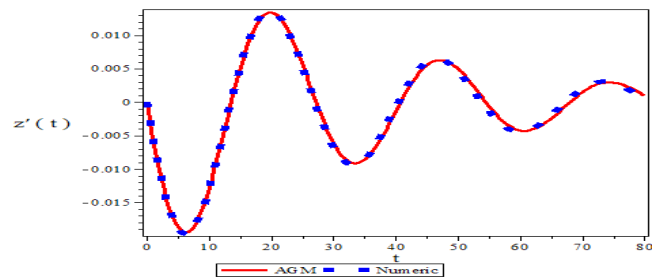AND NUMERICAL METHOD

FIG.8 THE RESULTED CHART OF VIBRATIONAL VELOCITY BY
AGM AND NUMERIC.

Difference the Obtained Solutions by AGM and Numerical Method

The following charts are the absolute valuedifference between the results on the basis of the yielded solution from Eq.(43) by AGM and the results of table.1 by Numerical Method:
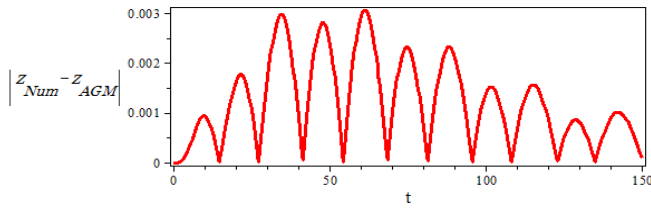
FIG. 9 DIFFERENCE THE OBTAINED SOLUTIONS BY AGM AND NUMERICAL METHOD.
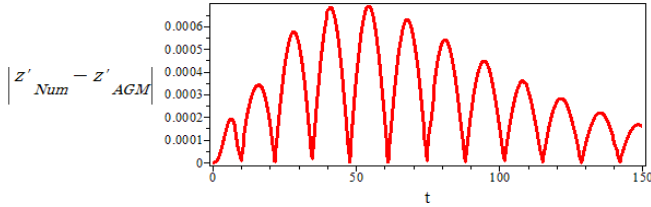

FIG. 10 DIFFERENCE THE FIRST DERIVATIVE OF THE OBTAINEDSOLUTIONS BY AGM AND NUMERICAL METHOD

Computation of Other Damping Vibrational Parameters

The maximum amplitude of vibrational displacement, velocity and acceleration equations can be computed as follows:

On the basis of the given explanations in part 2, the answer of the introduced differential equation is considered as:

$$z(t) = be^{-at}\{\cos(\omega t + \varphi)\} \tag{44}$$

The curve of locus for the maximum points of vibrational amplitudes can be achieved from Eq.(44) when $\cos(\omega t + \varphi) = \pm 1$, therefore we will have:

$$z_{\max}(t) = b\,e^{-at} \tag{45}$$

In the above equation, the constant coefficients a and b can be obtained on the basis of the given explanations. By substituting $t_k = \dfrac{k\pi}{\omega}$ in Eq.(45) which $k \in \{1,\,2,\,3,...\}$, it is possible to gain the maximum amplitudes such as $z_{1\max}$, $z_{2\max}$ and $z_{3\max}$. According to the given physical values, the maximum amplitudes can be obtained as follows:

$$z_{k\max} = b.e^{-a(\frac{k\pi}{\omega})} \tag{46}$$

Therefore, we will have:

$$k = 1 \quad \rightarrow \quad t_1 = \frac{\pi}{\omega} \quad \text{then} \tag{47}$$

$$z_{1\max} = b.e^{-a(\frac{\pi}{\omega})} = 0.100722\,e^{-0.02777\,(\frac{\pi}{0.230712})} = 0.069$$

$$k = 2 \quad \rightarrow \quad t_2 = \frac{2\pi}{\omega} \quad \text{as a result} \tag{48}$$

$$z_{2\max} = b.e^{-a(\frac{2\pi}{\omega})} = 0.100722\,e^{-0.02777\,(\frac{2\pi}{0.230712})} = 0.0472$$

The damping factor in the vibrational solution of $b\{e^{-at}.\cos(\omega t + \varphi)\}$ is e^{-at} since in the vibrational

differential equation the term $e^{-\xi\omega t}$ is the main factor for damping, (ξ) can be obtained as follows:

$$e^{-at} \equiv e^{-\xi\omega t} \quad \mapsto \quad \xi = \frac{a}{\omega} \tag{49}$$

In Eq(49) a and ω Are evaluated by applying boundary conditions.

And the initial vibrational phase can be obtained as follows:

$$\varphi = tg^{-1}(\xi) \tag{50}$$

In Eq.(50), the parameters a and ω have been gained in accordance with the given physical values. Therefore in this case study, it is clear that $\xi = 0.060091$. It is possible to simultaneously depict the solution of the differential equation and the chart of locus for maximum displacements as:
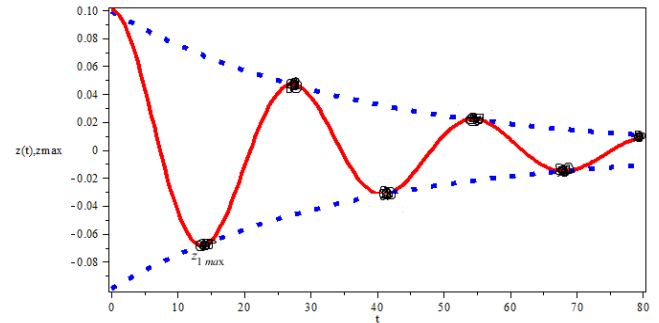

FIG. 11 THE CHARTS OF THE OBTAINED SOLUTION AND THE LOCUS OF MAXIMUM DISPLACEMENT POINTS

Furthermore, the maximum velocity $\dot{z}_{\max}$ can be acquired after taking the first derivative of Eq.(44) as follows:

$$\dot{z}(t) = b\{-a.e^{-at}.\cos(\omega t + \varphi) - e^{-at}.\omega.\sin(\omega t + \varphi)\} \tag{51}$$

As regards trigonometric relations, the maximum velocity ($\dot{z}_{\max}(t)$) from Eq.(51) can be obtained as follows:

$$\dot{z}_{\max}(t) = b\sqrt{a^2 + \omega^2}\,e^{-at} \tag{52}$$

In Eq.(52), t is defined as $t = t_k = \dfrac{(2k-1)\pi}{2\omega}$ which $k \in \{1,\,2,\,3,...\}$. As a result, the equation of maximum points of vibrational velocity is expressed as follows:

$$V = \dot{z}_{k\max} = b\sqrt{a^2 + \omega^2}\,e^{-a\frac{(2k-1)\pi}{2\omega}} \tag{53}$$

It is necessary to mention that a, b and ω have been obtained with respect to the given physical values. Consequently, the maximum vibrational velocities can be computed as follows:

$$k = 1 \rightarrow \dot{z}_{1\max} = 0.017157\,(\frac{m}{\sec})\ \text{and}$$

$$k = 2 \rightarrow \dot{z}_{2\max} = 0.01175\,(\frac{m}{\sec}) \tag{54}$$

Then, the chart of locus for vibrational velocity and the

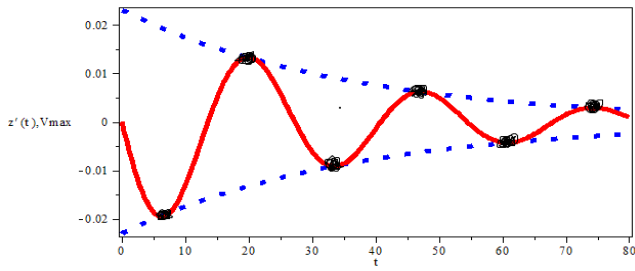own vibrational velocity chart are illustrated as:

FIG. 12 THE ACHIEVED RESULTS FOR THE VIBRATIONAL VELOCITY AND THE LOCUS OF THE MAXIMUM VIBRATIONAL VELOCITY POINTS

Moreover, the desired equation for vibrational acceleration will be obtained by computing the second derivative of the obtained solution or the first derivative of the velocity equation. And also by utilizing trigonometric equations, it is possible to compute the locus for maximum acceleration points in $t_k = \dfrac{k\pi}{\omega}$ as follows:

The locus of maximum acceleration points

$$\rightarrow \quad \ddot{z}(t) = b.(a^2 + \omega^2).e^{-at} \qquad (55)$$

The maximum acceleration point

$$\rightarrow \quad \ddot{z}_{k\,\max} = b.(a^2 + \omega^2).e^{-a(\frac{k\pi}{\omega})} \qquad (56)$$

In regard to the given physical values, the maximum vibrational accelerations in damping states can be obtained as follows:

$$k = 1 \rightarrow \ddot{z}_{1\max} = 0.00367 \ (\frac{m}{s^2}) \text{ and for}$$
$$k = 2 \ \rightarrow \ \ddot{z}_{2\max} = 0.002515 \ (\frac{m}{s^2}) \qquad (57)$$

Therefore, the charts of locus for maximum acceleration points and the obtained vibrational acceleration equation are depicted as follows:
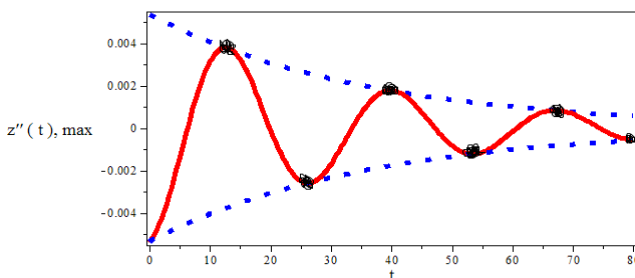

FIG. 13 THE RESULTED CHARTS OF THE VIBRATIONAL ACCELERATION EQUATION AND ITS RELATED LOCUS.

Results and Discussion

The chart of the angular frequency (ω) from Eqs.(40) in terms of initial amplitude of vibration in accordance with the introduced variables from Eq.(43) is illustrated as follows:
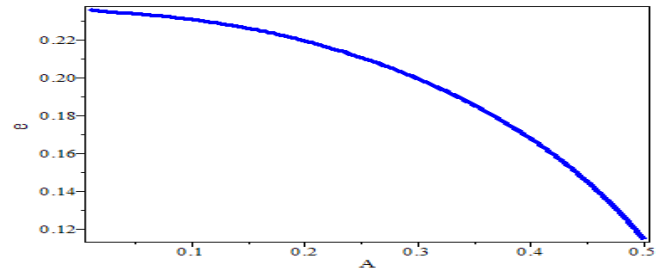

FIG. 14 THE CHART OF ANGULAR FREQUENCY IN TERMS OF INITIAL VIBRATIONAL AMPLITUDE.

It is clear that the more amount the amplitude of vibration in the initial condition (IC) is, the more decreasing of the angular frequency is.

The chart of damping ratio (ξ) in terms of initial amplitude of vibration (A) from Eq.(49) is depicted as follows:
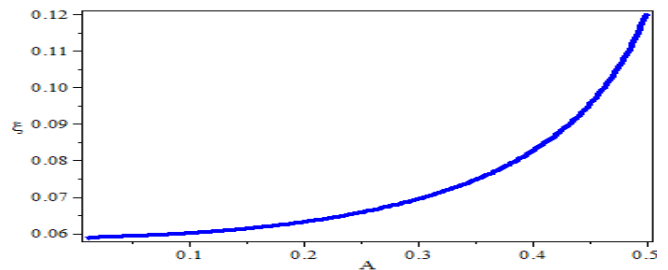

FIG. 15 THE VARIATION OF DAMPING RATIO IN TERMS OF THE INITIAL AMPLITUDE OF VIBRATION (A).

With regard to the above figure, it is revealed that by increasing the amount of initial amplitude of vibration (A), the value of damping ratio will be increased. Therefore in this step, it is logical to indicate that inasmuch as the formula of damping coefficient is obtained by $c = 2m\omega\xi$, there is a direct relationship between initial amplitude of vibration and damping coefficient.

Comparing the Charts of Phase Planes for Different Values of Amplitude of Vibration by AGM

For various amounts of vibrational amplitudes, take for example A=0.1, 0.18, 0.26 and finally A=0.34, the resulted phase planes are illustrated by AGM as:
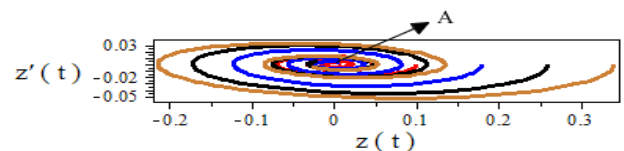

FIG. 16 COMPARING THE OBTAINED PHASE PLANES BY AGM ON THE BASIS OF INCREASING THE VALUES OF AMPLITUDE OF VIBRATION (A)

Assuming hinged-hinged to beam-column, shape function is defined as follows:

$$\phi_n(x) = \phi_0 \sin(\frac{n\pi x}{L}) \qquad (58)$$

Resulting in the displacement space-time is calculated as follows:

$$U_n(x,t) = z(t) \cdot \phi_0 \sin(\frac{n\pi x}{L}), \ n = 1,2,3,... \qquad (59)$$

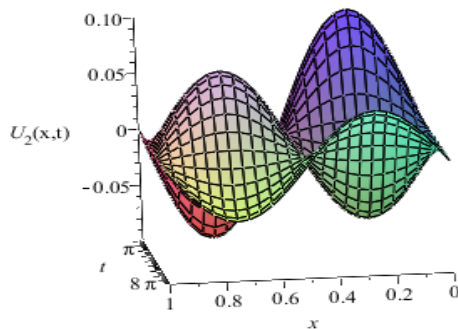Displacement curve is plotted in the form of three-dimensional coordinates:

FIG. 17 DISPLACEMENT CURVE SPACE - TIME MODE n=2 VIBRATIONAL.

Conclusions

In this paper, complicated nonlinear vibrational partial differential equations have been introduced and analyzed in details by Algebraic Method (AGM) and also the obtained results have been compared with Numerical Method(Runge-Kutte 4th). Then, the vibrational velocity and vibrational acceleration have successfully been achieved. Afterwards, the related equations of locus for vibrational velocity and acceleration have been gained and depicted completely. Eventually, the equation of damping ratio in terms of initial amplitude of vibration and angular frequency has been obtained perfectly. The above process has been done in order to show the ability of AGM for solving a broad range of differential equations in different fields of study particularly in vibrations. Consequently, it is concluded that AGM is a reliable and precise approach for solving miscellaneous differential equations. Moreover, a summary of the AGM excellence and benefits is explained as: By solving a set of algebraic equations with constant coefficients, we are able to obtain the solution of nonlinear differential equation along with the related angular frequency simultaneously very easily as applying this procedure is possible even for students with intermediate mathematical knowledge. On the other hand, it is better to say that AGM is able to solve linear and nonlinear differential equations

directly in most of the situations, which means the final solution can be obtained without any dimensionless procedure. Therefore, AGM can be considered as a significant progress in nonlinear sciences. Inasmuch as the shortage of boundary condition(s) in this method for solving differential equation(s) is completely terminated, AGM is operational for miscellaneous nonlinear differential equations especially for the vibrations of civil engineering and Mechanical machinery and it is prospected that this method will be applied by enthusiastic young researchers in the near future.

REFERENCES

D.D. Ganji, The application of He's homotopy perturbation method to nonlinear equations arising in heat transfer, Phys. Lett. A 355, 337–341, 2006.

S.P Timoshnko, strength of material, part H, 3rd ed. (New-York: Van Nostrand Company, 1956.

Chopra, Anilk, "Dynamic of structural theory and applications to earthquake engineering" 1995. Prentice-Hall, Inc. Englewood Cliffs, New Jersey 07632.

Clough.R.W. and penzien, J. "Structural Dynamic". McGraw. Hill. New York. 1975.

M. G. Sfahani, A. Barari, M. Omidvar, S. S. Ganji, G. Domairry, Dynamic response of inextensible beams by improved energy balance method, Proceedings of the Institution of Mechanical Engineers, Part K Journal of multi-body Dynamics, 225 (1), 66-73, 2011.

Naeim,F., "The Seismic Design Handbook", VanNastad Ed ,1986.

S. S. Ganji, D. D. Ganji, H. Babazadeh, N. Sadoughi,

Application of amplitude-frequency formulation to nonlinear oscillation system of the motion of a rigid rod rocking back, Mathematical Methods in the Applied Sciences, 33 (2), 157–166,2010.

Walter Lacarbonara, A Theoretical and Experimental Investigation of Nonlinear Vibrations of Buckled Beams. Master of Science Thesis, Virginia Politechnic Institute and State University (1997).

W. Lestari, S. Hanagud, Nonlinear vibration of buckled beams: some exact solutions, International Journal of Solids and Structures, 38, 4741-4757 (2001).

F.S. Tse, I.E. Morse, R.T Hinkle, Mechanical Vibrations: Theory and Applications. Second ed. Allyn and Bacon Inc.,bosto (1978).

Xu-Hong WU and Ji-Huan He. "Solitary solutions, periodic solutions and compacton-like solutions using the Exp-function method "Computers & mathematics with applicatios, Volume54, Issue 7-8, 966-986, 2007.

Abdoul R. Ghotbi, H. Bararnia, G. Domairry, A. Barari, Investigation of a powerful analytical method into natural convection boundary layer flow, Commun Nonlinear SciNumerSimulat , 14, 2222–2228 (2009).

A. Kimiaeifar, A.R. Saidi, G.H. Bagheri, M. Rahimpour, D.G. Domairry, Analytical solution for Van der Pol–Duffing oscillators, Chaos, Solitons and Fractals, 42, 2660–2666 (2009).

S.S. Ganji, D.D. Ganji, Z.Z. Ganji, S. Karimpour, Periodic solution for strongly nonlinear vibration system by He's energy balance method, ActaApplicandaeMathematicae, doi:10.1007/s10440-008-9283-6 (2008).

Ji-Huan He and Xu-Hong Wu. "Exp-function method for nonlinear wave equations" Chaos. Solitos & Fractals. Volume 30, Issue 3, November 2006, 700-708, 2006.

T.B Pekoz, "Torsional-flexural buckling of thin-walled section under eccentric load," cornell engineering research bulletin, No 69-1, Ithaca, N,Y., 1969.

J.H. He, Homotopy perturbation technique, J. Comput. Methods Appl. Mech. Engrg.17 (8), 257–262, 1999.

A. Kimiaeifar, A. R. Saidia, A. R. Sohouli, D. D. Ganji, Analysis of modified Van der Pol's oscillator using He's parameter-expanding methods, Curr. Appl. Phys., 10 (1), 279-283, 2010.

Thomason William T. "Theory of vibration with applications", Pentice. Hall, Inc, Englewood cliffs, Newjersey, 1972.

Bolt, B.A., "Duration of strong motions", Proc 4th Wold Conf.Earthq.Eng.,Santiago,Chile,1969,pp 1304-1315.

M. Momeni, N. Jamshidi, A. Barari, D. D. Ganji, Application of He's energy balance method to duffing harmonic oscillators, International Journal of Computer Mathematics, 88, 135–144, 2011.